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Peter Bodenheimer, John C. Brandt, and R.R. Robbins

Space Sciences Laboratory
University of California
Berkeley 4, California

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INTERPLANETARY GAS. VIII. ON THE ROLE OF RADIATIVE LOSSES[1]

Peter Bodenheimer, John C. Brandt, and R. R. Robbins
Berkeley Astronomical Department, The Space Sciences Laboratory
University of California

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Abstract

The radiative losses due to free-free transitions, free-bound transitions, and permitted and forbidden line emission have been evaluated for all ions which are present in significant amounts in the extended solar corona from $4R_{\odot}$ to $215R_{\odot}$. Most of the radiative losses occur interior to $15R_{\odot}$, and they are dominated by line emission in the extreme ultraviolet and x-ray wavelengths. In general, the radiative losses are found to be unimportant, and the consequences of this result are discussed in terms of the semiempirical model recently presented by Brandt and Michie (1962).

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1. Introduction

The determination of the radiative losses in the solar corona and extended solar atmosphere is of importance in discussions of the energy balance and structure of the corona. Earlier estimates of these losses have been made (Woolley and Allen 1948; Elwert 1954; Zirin 1957; Allen 1961; Lüst and Zirin 1960; De Jager and Kuperus 1961). However the observational values of many of the physical quantities in the corona and interplanetary medium have not been made certain as yet. As work progresses and more observational material is obtained, the calculation of the radiative losses which are based on this data must continually be revised.

In a recent paper, Brandt and Michie (1962, Paper VII) obtain a theoretical expression for an expanding solar corona which they find to be reasonably consistent with empirical evidence (Brandt 1962a); recently, the values of the expansion velocity and density found by Brandt and Michie (1962) have been confirmed by the results obtained from Mariner-II by Neugebauer and Snyder (1962). In such an expanding corona, if $\mathcal{F}_e$ and $\mathcal{F}_m$ are, respectively, the total energy per second and the total mass per second which cross a spherical surface at a distance r from the sun, then at a point beyond which energy deposition and radiative losses can be neglected, the ratio $\mathcal{F}_e / \mathcal{F}_m$ will be a constant independent of r. Brandt and Michie have evaluated this constant at 4 solar radii, thereby introducing the assumption that radiative losses and energy deposition are negligible beyond that point. It is the purpose of this paper to test the validity of the assumption with regard to the radiative losses.

2. Preliminary Results

In order to find the rate of energy lost by radiation in the entire region between 4 solar radii and the orbit of the earth, we must evaluate an integral of the form

$$T = 4\pi \int_{4R_\odot}^{215R_\odot} \epsilon(r) r^2 dr \qquad \text{erg/sec} \tag{1}$$

where ϵ (r) is the rate of energy loss per cm^3 by all radiative processes. The quantity $\epsilon(r)$ may be broken up into three contributions: continuous emission due to free-free transitions, continuous emission due to free-bound transitions, and line emission. These contributions must be evaluated for all ions which are present in significant amounts in the extended solar corona.

The abundance of the various ions is determined first from an assumption concerning the abundances of the elements in the region under consideration and second from a calculation of the state of ionization of the various elements. We base our calculations upon the relative abundances of the elements as given by Goldbert, Muller, and Alley (1960); the elements considered and their relative abundances are indicated in Table 1.

To determine the state of ionization of the various elements we note that the ionization equilibrium in the corona is established by a balance between radiative recombinations and excitation by collisions with electrons, under the assumption that photo-ionizations and three-body recombinations can be neglected (Woolley and Allen 1948). However the results of Brandt and Michie (1962) indicate that the gas of the corona is expanding outwards with velocities ranging from about 8 km/sec at 4 solar radii to 300 km/sec at the orbit of the earth. Because of this expansion, the material at a given point may not have had time to reach secular equilibrium between the various stages of ionization.

Consider two points, A and B. The time for material to travel from A to B is

$$t_j = D_{AB}/\langle v \rangle_{AB} \tag{2}$$

where D_{AB} is the distance from A to B and $\langle v \rangle_{AB}$ is the average velocity between A and B. If the ions in stage of ionization p recombine and no new ions in stage p are produced, then the time for the number of ions in stage p to drop to 1/e of its original value is

$$t_r = \langle \alpha \, N_e \rangle^{-1} \tag{3}$$

where N_e is the electron density and α is the recombination coefficient to stage p-1. The quantity t_r then is a characteristic time for recombination of ions when the material moves to a region favoring lower ionization. A sufficient condition that the relative number of ions in a particular stage not change when the material moves from A to B is

$$t_j \ll t_r \tag{4}$$

We find, using the estimates of expansion velocity by Brandt and Michie, that for $A = 2R_\odot$ and $B = 4R_\odot$, $t_j = t_r/8$; and for $A = 4R_\odot$ and $B = 15R_\odot$, $t_j = t_r/20$ for an ion of charge $Z = 14$; furthermore t_j will be even smaller compared to t_r for ions of a smaller charge. Therefore we conclude that the state of ionization of the elements in the outer corona beyond 4 solar radii is the same as it is in the corona within 4 solar radii.

In the corona we may assume that secular equilibrium holds, that is, at a given point the rate of formation of ions of stage p equals the rate of disappearance of ions from stage p. An ion in stage p can be created through recombination from stage p + 1 or through ionization from stage p - 1. Under the assumption that three-body recombinations and photoionizations can be neglected, we arrive at the secular equilibrium equation

$$N_e N_p (S_{p \to p+1} + \alpha_{p \to p-1}) = (N_{p-1} S_{p-1 \to p} + N_{p+1} \alpha_{p+1 \to p}) N_e \tag{5}$$

where N_e represents the electron density, N_p, N_{p-1}, and N_{p+1} represents the ion densities of stages p, p-1, p+1, respectively, $S_{i\rightarrow j}$ represents the collisional ionization coefficient for stage i, and $\alpha_{i\rightarrow j}$ represents the recombination coefficient onto stage j. If stage p corresponds to a completely ionized atom, equation (5) reduces to

$$N_p \alpha_{p\rightarrow p-1} = N_{p-1} S_{p-1\rightarrow p} \tag{6}$$

Now, by replacing p by p-1 in equation (5) and by substituting equation (6) into the result, we obtain another equation of the same form as (6). Thus we find that for each stage of ionization the following relation (due to Woolley and Allen 1948) holds:

$$\frac{x}{1-x} = \frac{S_{p\rightarrow p+1}}{\alpha_{p+1\rightarrow p}} \tag{7}$$

where (Woolley and Allen 1948, Allen 1955b)

$$S_{p\rightarrow p+1} = 3 \times 10^{-8}\, T^{1/2}\, \chi_i^{-2} \exp\left(\frac{-11600\chi_i}{T}\right) , \tag{8}$$

$$\alpha = 1.5 \times 10^{-11}\, Z^2\, T^{-1/2} , \tag{9}$$

Z is the ionic charge of stage p+l, χ_i is the ionization potential of stage p in ev, and x is the number of ions in the p+1 stage divided by the total number of ions in the p and p+1 stages.

From equation (7) we calculate the ion densities of the more abundant species relative to hydrogen, assuming a temperature of $10^{6\,o}K$ and using the cosmic abundances from Table 1. The results are shown in

Table 2. The method is not used for the elements hydrogen and helium; we assume that both elements are essentially completely ionized in the region under consideration.

The calculation of radiative loss also requires a knowledge of the temperature and electron density distribution in the outer corona. The temperature distribution near the sun can be obtained from the observed densities by use of the assumption of hydrostatic equilibrium (Pottasch 1960, Chapman 1961). We use two different temperature distributions, both of which agree with the Chapman values between 4 and 5 solar radii. Distribution (A) uses a probable upper limit of $100,000^{\circ}K$ for the temperature at 1 A. U., while distribution (B) uses a probable lower limit of $30,000^{\circ}K$ at that point (Chamberlain 1961). The intermediate values are obtained by interpolation.

Two distributions of electron density are obtained in a similar manner. Between 4 and 6 solar radii the values in both cases are those observed by Blackwell (1956, see also van de Hulst, 1950). Near the orbit of the earth the value $N_e = 1$ (Brandt 1962a, Ingham 1961) is used in distribution (a) while in distribution (b) the value $N_e = 4$ is used as a reasonable upper limit. The intermediate values are again interpolated. Since we take the ratio of abundances of hydrogen to helium as 10:1 and since both are completely ionized, the density of hydrogen ions is $N_i = 0.83N_e$.

In our calculations we shall assume a Maxwellian velocity distribution, which is a reasonable approximation at points close to the sun but which breaks down at large distances. Fortunately the calculations show that most of the radiative loss occurs at distances of less than 15 solar radii, therefore the introduction of this assumption should not result in appreciable error.

3. The Method of Calculation

We now turn to the actual calculation of the quantity $\epsilon(r)$. The intensity of the free-free emissions is given by (Allen 1955a)

$$\epsilon_{ff} = 1.44 \times 10^{-27}\, gT^{1/2}\, Z^2\, N_e N_i \qquad \text{ergs/cm}^3\text{sec} \tag{10}$$

where g is of the order unity, Z is the ionic charge, N_e and N_i are the numbers of electrons and ions, respectively, per cm^3. Using the fact that for a given ion

$$N_i = 0.83\, N_e p_i \tag{11}$$

where p_i is the abundance of the ion relative to hydrogen, we obtain

$$\epsilon_{ff} = 1.19 \times 10^{-27}\, T^{1/2} N_e^{\,2} \sum_i Z_i^{\,2} p_i \qquad \text{ergs/cm}^3\text{sec} \tag{12}$$

where the summation is over all the ions of interest. Calculation of the quantity $\sum_i Z_i^{\,2} p_i$ gives a result of 1.4 for hydrogen and helium and 0.076 for all the other elements. Thus the final expressions become

$$\left.\begin{aligned} \epsilon_{ff}(H + He) &= 1.67 \times 10^{-27}\, T^{1/2}\, N_e^{\,2} \\ \epsilon_{ff}(\text{total}) &= 1.76 \times 10^{-27}\, T^{1/2}\, N_e^{\,2} \end{aligned}\right\} \frac{\text{ergs}}{\text{cm}^3\text{sec}} \tag{13}$$

The rate of free-bound emission is derived by Menzel (1937); the total energy emitted in transitions from the continuum to a level n of a hydrogen-like atom is

$$\epsilon_{fb} = N_i N_e k K Z^4 g b_k / n^3 T^{1/2} \qquad \text{ergs/cm}^3\text{sec} \tag{14}$$

where b_k indicates the departure from a Maxwellian velocity distribution. Summing over n, evaluating the constants, setting $b_k = 1$, and making use of equation (11) we have

$$\epsilon_{fb} = 4.52 \times 10^{-22} N_e^{2} T^{-1/2} \sum Z^4 p_i \quad \text{ergs/cm}^3\text{sec} \tag{15}$$

This formula is applicable to the calculation of the free-bound energy loss for H II and He III only; for these we find that $\sum Z^4 p_i = 2.6$. For the heavy elements we use equation (21) below.

In the calculation of the bound-bound energy loss we first note that at the dilute radiation densities and low particle densities under consideration, the times for particle or photon excitation or de-excitation of bound electrons are much greater than spontaneous radiation times ($\sim 10^{-8}$ sec). Thus an electron in an excited state will, in general, cascade to the ground state before any other process can occur. Therefore part of the bound-bound emission will be due to electrons which cascade to the ground state after they arrive at levels n = 2 or higher by processes of recombination. To find the energy from this source we multiply the total number of recombinations to each level by the energy difference between that level and the ground level. The recombination coefficient to the nth level of a hydrogen-like atom is given by Cillie (1932) as

$$\alpha_n = 3.26 \times 10^{-6} n^{-3} T^{-3/2} Z^4 \quad \exp\left\{\chi_n/kT\right\} \quad E_1(\chi_n/kT) \tag{16}$$

where $$E_1(x) = \int_x^\infty \frac{e^{-t}}{t} dt \tag{17}$$

and where χ_n for a hydrogen-like atom is given by

$$\chi_{n}/kT = \frac{157900 Z^2}{n^2 T} = Q \quad . \tag{18}$$

The energy is then

$$\epsilon_{bb(1)} = \sum_{n=2}^{\infty} (\chi_i - \chi_n)\; a_n N_i N_e \quad \text{ergs/cm}^3\text{sec} \tag{19}$$

where χ_i is the ionization potential. This formula is used in the calculations in the case of hydrogen and helium. For the heavy elements we estimate the energy loss due to free-bound transitions and subsequent cascades to the ground level by using the generalized recombination coefficient of a non-hydrogen-like atom, given by Allen (1955b):

$$a \cong 1.5 \times 10^{-11} Z^2 T^{-1/2} \quad . \tag{20}$$

The energy loss becomes

$$\epsilon_{fb + bb(1)} \left[\text{heavy elements} \right] = a\, N_e N_i (\bar{\epsilon} + \chi_i) \tag{21}$$

where $\bar{\epsilon}$ is the average kinetic energy of the recombining electron, assumed to be 100 ev. This formula is used for all ions listed in Table 2.

The second source of bound-bound emission is collisional excitation by electrons from the ground level followed by cascade back to the ground level. For the case of hydrogen and helium we follow the method of Zirin (1957). To determine the energy due to bound-bound transitions he assumes that ionization-recombination equilibrium holds:

$$\text{N(H I) x ionizations} = \text{N(H II) x recombinations.} \tag{22}$$

However our calculation is based upon a model of an expanding corona; therefore we would not expect such an equilibrium to hold. Since hydrogen is completely ionized in the inner corona, the number of H II ions at any point in the outer corona is actually greater than the number one would expect under conditions of equilibrium. Our calculation of the bound-bound energy from this source therefore represents an upper limit since it is based on the equilibrium assumption.

The hydrogen line emission is given by Zirin (1957, eq. 4A):

$$\epsilon_{bb(2)} = N(\mathrm{H\ II}) \times \frac{\text{recombinations}}{\text{ionizations}} \times \text{excitations} \times \chi_i \quad . \tag{23}$$

A similar equation holds for helium. However, Bethe (1933a) shows that for hydrogen-like ions, the rate of collisional excitation to all levels and the rate of collisional ionization are in the ratio of about 54:36 for electron energies of 1000 electron volts, which corresponds to an electron temperature of 10^7 °K. Furthermore, extrapolation of Bethe's table indicates that the ratio tends towards 1 for lower electron energies. Therefore we can take as valid Zirin's assumption that the rates of collisional ionization and collisional excitation are roughly equal in the case of hydrogen. Bethe also shows (1933b) that the rates of collisional ionization and excitation both depend on Z^2, so the ratio is independent of Z and the same assumption can be applied in the case of helium. We use the following expression in the determination of the bound-bound energy loss from hydrogen and helium, based upon a formula from Cillie (1932):

$$\epsilon_{bb(2)} = 3.26 \times 10^{-6} N_e N_i \quad \chi_i Z^4 T^{-3/2} \quad \sum_{n=1}^{\infty} \frac{\exp(Q) E_1(Q)}{n^3} \tag{24}$$

where χ_i is the ionization energy and Q is given by equation (18).

We now turn to the calculation of the radiative loss due to permitted lines of the heavy elements. Because $1/A \gg t_j$ for distances over which T and N_e change, secular equilibrium for excitation is valid

at any point, and the number of bound electrons collisionally excited in a given time interval equals the number of photons produced. Following the development of Woolley and Allen (1948), the energy loss per cm^3 per sec in a given multiplet whose lower term is the ground term is

$$\epsilon_{\text{permitted}} = \frac{1.88 \times 10^{-15} N_e^{\;2}}{T^{1/2}} f_{em} p_i \left\{ \exp(-y) - y E_1(y) \right\} \tag{25}$$

where $y = \chi'/kT$, χ' is the excitation potential of the multiplet, and f_{em} is the absolute value of the emission f-value for the transition. The f-values used in the calculations are those recently computed by Varsavsky (1961).

A number of important permitted lines fall in the X-ray region, primarily below 100 A; f-values for these transitions are not given by Varsavsky. In order to take these into account we make an estimate based on the calculations of Elwert (1954, 1958). He gives (1958, eq. 12) the energy loss resulting from collisional excitation per cm^3 per sec as

$$J^L = \epsilon(\text{X-ray}) = f_3 \cdot 4 \times 10^{-19} \left(\frac{10^6}{T} \right)^{1/2} N_e^{\;2} \sum Y_{zi} \tag{26}$$

where the summation is taken over all the elements and stages of ionization under consideration; x-ray lines considered are listed in Table 2. Here, f_3 is an uncertainty factor of the order unity and the quantities Y_{zi} include abundance factors as well as collisional cross sections. The ions, the wavelengths of the lines considered, and the Y_{zi} as a function of temperature are listed in Elwert (1954), Table 5. The calculation was made with our electron density distribution and an assumed $T = 6 \times 10^5$ °K, a simplification which can introduce only negligible error since the quantity J^L turns out to be only weakly dependent on temperature. We note that two lines in Elwert's table, λ 40 of C V and λ33 of C VI, are also included in our

calculation of equation (25). A comparison of the radiative loss due to these lines, based on the two independent methods of calculation, shows reasonable agreement; therefore we feel justified in using Elwert's formula for calculating the X-ray emission.

In turning to the calculation of the radiative loss from forbidden lines, we first note that only photons arising from collisional excitation will affect the flux balance. Unfortunately, the excitation cross section has been calculated for only one line, the λ5303 line of Fe XIV, in which case the excitation cross section is given by

$$\sigma(v) = 0.78\ v^{-2}\ \mathrm{cm}^2 \tag{27}$$

It follows that the collisional excitation rate from level a to level b is (Firor and Zirin, 1962)

$$C_{ab} = 1.60 \times 10^{-6}\ T^{-1/2} N_e \tag{28}$$

If we assume, as do Firor and Zirin, that the excitation cross sections for all the forbidden lines under consideration are about the same, we may estimate the emission due to forbidden lines by the formula

$$\epsilon_{\mathrm{forbidden}} = 1.33 \times 10^{-6}\ T^{-1/2}\ N_e^{\ 2} \sum_i p_i h\nu_i \quad \mathrm{ergs/cm^3 sec} \tag{29}$$

In the calculations, all of the forbidden lines listed in Table II of Woolley and Allen (1948) were considered.

To summarize, the formulae used in the calculations are (13), (15), (19), (21), (24), (25), (26), and (29). Substituting $N_i = 0.83\ p_i N_e$ into equations (19), (21), and (24), evaluating $\sum h\nu_i p_i$ in equation (29) and $\sum Z^2 p_i(\bar{\epsilon} + \chi_i)$ in equation (21), using $\sum Y_{zi} = 27 \times 10^{-6}$ in equation (26), including an estimate of the bound-bound contribution from upper levels of the heavy atoms in equation (25) (see Results), and combining terms where possible, we arrive at the following expression for the total emission per cm^3 per sec:

$$\frac{\epsilon_{tot}}{N_e^{\,2}\,\mho X} = 1.76 \times 10^{-27} T^{1/2}$$

$$+ \left[1.28 \times 10^{-20} + 2.82 \times 10^{-15} \sum_{\substack{\text{heavy}\\ \text{elements}}} f_{em} p_i \left\{ \exp(-y) - y E_1(y) \right\} \right] T^{-1/2}$$

$$+ 2.70 \times 10^{-6} T^{-3/2} \left(\sum_{\substack{n=1 \\ i=\mathrm{H,\,He}}}^{\infty} Z^4 p_i \chi_i \frac{\exp(Q) E_1(Q)}{n^3} \right.$$

$$\left. + \sum_{\substack{n=2 \\ i=\mathrm{H,\,He}}}^{\infty} (\chi_i - \chi_i) \frac{Z^4 p_i}{n^3} \exp(Q) E_1(Q) \right) \tag{30}$$

where

$y = \frac{\chi'}{kT}$ and χ' is the excitation potential for a heavy element

$$Q = \frac{157900\, Z^2}{n^2 T}$$

χ_i = ionization potential

$\chi_i - \chi_n$ = excitation potential for H and He II

p_i = abundance of ion with respect to hydrogen

Z_i = ionic charge

$\mho \sim 2$, uncertainty factor explained in section 5

$1/X \sim 1/3$, fraction of space occupied by matter, explained under Results (5).

Evaluating equation 30 for the case of a constant temperature of $T = 10^6\ ^\circ K$ we arrive at the following expression:

$$\frac{\epsilon_{tot}}{N_e^{\;2}UX} = 2.25 \times 10^{-23} \quad \text{erg cm}^3/\text{sec} \quad . \tag{31}$$

With the various ionic abundance as calculated for T = 1×10^6 °K but with T = 7.5×10^5 °K we obtain

$$\frac{\epsilon_{tot}}{N_e^{\;2}UX} = 2.34 \times 10^{-23} \text{ °K erg cm}^3/\text{sec} \tag{32}$$

The individual sources of radiative loss have been computed separately, and the results are shown in Table 3. The total has been checked by substituting equation (32) into equation (1).

4. Results

In summarizing the radiative losses due to the various processes described above (see Table 3) we make the following observations:

(1) The overwhelming contribution to the radiative loss occurs within 15 solar radii. As a result the energy losses differ very slightly when different models of temperature and electron density are employed, since the models differ significantly from one another only at large distances from the sun.

(2) The single most important source is the X-ray lines which contributes a little over half the total.

(3) The heavy ion which makes the largest contribution to the radiative loss in the ultraviolet region is Si XI. However, it is probably not safe to give strong weight to the calculations for individual lines or ions.

(4) The total radiative loss from the heavy elements in the ultraviolet region is about 9×10^{21} ergs/sec. However, in the calculations only the transitions from the first excited level to the ground level were accounted for. Transitions between the higher levels, with subsequent cascades to the ground level, were not calculated since no reliable f-values are available for such transitions. However, in order to estimate the relative importance

of the higher levels, we may look back at the calculations for H and He. Here the calculations did take into account the upper-level transitions and subsequent cascade to the ground level; such transitions constituted about 1/3 of the bound-bound radiative loss for H I and He II. If it is also true that 1/3 of the radiative loss from bound-bound transitions in heavy elements comes from the upper levels, then the total radiative loss in the ultraviolet from the heavy elements should be 13.5×10^{21} ergs/sec instead of 9×10^{21} ergs sec.

(5) The total, computed radiative loss from all sources is $\sim 4.9 \times 10^{22}$ ergs/sec. However we must note here that actually $\langle Ne \rangle^2 = 1/X \langle Ne^2 \rangle$ where 1/X can be thought of as the fraction of space occupied by matter (see Unsöld 1960 and Brandt 1962b). Thus we are underestimating the quantity N_e^2, which appears in every expression which we evaluate by the factor 1/X. The estimates of the quantity 1/X (valid near $5R_\odot$) are about 1/5 (Unsöld 1960). This correction would raise the radiative loss to about 2×10^{23} ergs/sec.

(6) The diffuse intensity of the permitted ultraviolet lines contributing to the radiative energy loss is very small ($4\pi\mathcal{J} < \sim 10$ R) for observations at elongations corresponding to distance from the center of the sun of $4R_\odot$ or greater.

5. Comparison with Other Data

To check the calculations we may compare them with the rocket observations of the solar flux per cm^2 in the ultraviolet and X-ray regions at the earth's distance from the sun. To carry out the comparison we integrate equation (1) backwards to the inner boundary of the corona ($r = 1.015\ R_o$) using the electron densities for solar minimum given by de Jager (1959). This is essentially a calculation of $\int N_e^2 dV$; we note that the same integral for the solar maximum data given by de Jager is 16 times the value for solar minimum. The comparison can at best be a rough one, principally because (1) the experimental results are uncertain by a factor of 3 to 5 and (2) and because of the uncertain role of the transition region in the production of ultraviolet photons.

Elwert (1961) quotes the measured values 0.13, 0.3, and 1.0 erg/cm^2sec for the minimum, mean, and maximum solar flux outside the earth's atmosphere in the X-ray region between 44 and 100 Å. His corresponding theoretical calculations give a mean value of 0.12 $\mho$ erg/cm^2

sec, where $\mho$ indicates the uncertainties in the theoretical cross sections. Elwert's calculation agrees with the observations if $\mho$ is taken between 2 and 4. Our calculations, on the other hand, give a result of 0.05 $\mho$ erg/cm^{2}sec; for solar minimum. We consider that only one-half of the X-rays produced escape into space, and that $X \approx 2$. Our calculations then require $\mho \approx 2$. Such a value is quite reasonable since $\mho$ must include errors in the observational results as well as in the theoretical cross sections, it also must include all sources of radiative loss which were not taken into account in the calculations (incompleteness).

Hinterreger (1961) has also published observations covering the ultraviolet and X-ray regions. His total observed flux due to the metallic lines in the region $\lambda < 1300$ A., corresponding in the most part to our calculated lines, is about 1.5 erg/cm^{2}sec. Our theoretical result (assuming $X \approx 2$ and that half the photons escape) is 0.08 $\mho$ erg/cm^{2}sec for solar minimum and 1.3 $\mho$ erg/cm^2-sec for solar maximum. Since Hinterreger's observations refer to solar maximum conditions (Cragg 1961), we have $\mho \approx 1$, i.e., satisfactory agreement.

Woolley and Allen, in their 1948 estimate of radiative losses from the corona, obtain a value for line emission in the ultraviolet which is about 10 times the value we have obtained for solar minimum. This figure is easily explained by the following two considerations: (1) use of the mean Baumbach-Allen densities raises their value by a factor of three and (2) lacking both observations and f-values for the far ultraviolet, they assumed all line emissions to be concentrated into ultimate lines for which f = 1, whereas the average f-value for lines in this region is much less than this (Varsavsky 1961). Our calculations are also in satisfactory agreement with the recent calculations of Allen (1960), when differences in assumed abundances and f-values are considered. We expect that our calculations are an improvement over Allen's (1961) because of the use of Varsavsky's (1961) f-values.

6. Consequences

Brandt and Michie (1962, eq. 20) derive an expression relating the conduction, velocity space anisotropy, expansion velocity, temperature, and heliocentric distance at points in the corona. The derivation of this expression is based on the assumption that $\mathcal{F}_e/\mathcal{F}_m$, the ratio of energy flow to mass flow across a spherical surface at distance r from the sun, is a constant, independent of r, beyond a certain distance which is assumed to be 4 solar radii. The derivation did not take into account radiative losses. We now re-derive the expression with the inclusion of a radiative loss term in order to demonstrate that the assumption made earlier still is, in fact, valid.

Equation (13) of Brandt and Michie (1962) for the energy flow is modified to become

$$\mathcal{F}_e = 4\pi r^2 \cdot (m/2)\left(\int_0^{\infty} \int_{-1}^{+1} \mu v^3 F \cdot 2\pi v^2 d\mu dv + 2Nw\langle v_r^2 \rangle + Nw\left[\langle v^2 \rangle + w^2 - \frac{2GM_{\odot}}{r} \right] \right) + 4\pi r^2 \mathcal{F}_{rad} \tag{33}$$

where $F(\vec{r}, \vec{v}, \mu)$ is the distribution function in terms of the peculiar velocity, $\vec{v}$ is the peculiar velocity vector, w is the mean expansion velocity, θ is the angle between the radius vector $\vec{r}$ and the peculiar velocity, $\vec{v}$, $\mu = \cos\theta$, $m = (1/2)m_H$ (the effective particle mass),

$$N = \int_0^{\infty} \int_{-1}^{+1} F(\vec{r}, \vec{v}, \mu) \cdot 2\pi v^2 d\mu\, dv \quad , \tag{34}$$

and $\mathcal{F}_{rad}$ is the radiative flux (The subscript r refers to the radial component.) Since the mass flow is given by

$$\mathcal{F}_m = 4\pi r^2 mNw \tag{35}$$

we have

$$\mathcal{F}_e/\mathcal{F}_m = (1/2)\left\{ \frac{1}{Nw} \int_0^{\infty}\int_{-1}^{1} \mu v^3 F \cdot 2\pi v^2 d\mu dv + 2 \langle v_r^2 \rangle + \langle v^2 \rangle \right. \tag{36}$$

$$\left. + w^2 - \frac{2GM_{\odot}}{r^2} \right\} + \frac{\mathcal{F}_{rad}}{mNw} \quad .$$

Let

$$\langle v^2 \rangle = \frac{3kT}{m} \tag{37}$$

$$\gamma = 3\left[\frac{\langle v_r^2 \rangle}{\langle v^2 \rangle} + 1/2 \right] \tag{38}$$

$$I = \frac{1}{Nw} \int_0^{\infty}\int_{-1}^{+1} \mu v^3 F \cdot 2\pi v^2 d\mu dv \tag{39}$$

and (36) modifies to

$$\frac{\mathcal{F}_e}{\mathcal{F}_m} = (1/2)\left\{ I + w^2 - \frac{2GM_{\odot}}{r} + \frac{\gamma kT}{m} \right\} + \frac{\mathcal{F}_{rad}}{mNw} \tag{40}$$

Evaluating the quantity $\mathcal{F}_e/\mathcal{F}_m$ at some inner position r_o we obtain

$$\frac{I_o}{2}+\frac{w_o^{\,2}}{2}-\frac{GM_\odot}{r_o}+\frac{kT_o\gamma_o}{m}+\frac{\mathcal{F}_{rad,\,o}}{mN_ow_o}=\frac{I}{2}+\frac{w^2}{2}-\frac{GM_\odot}{r}$$

$$+\frac{kT\gamma}{m}+\frac{1}{mNw}\left(\mathcal{F}_{rad,\,or}+\mathcal{F}_{rad,\,r_or}\right) \tag{41}$$

where $\mathcal{F}_{rad,\,or}$ is the radiative flux originating below r_o and $\mathcal{F}_{rad,\,r_or}$ is the radiative flux originating between r_o and r. Noting that

$$\frac{\mathcal{F}_{rad,\,o}}{mN_ow_o}=\frac{\mathcal{F}_{rad,\,or}}{mNw} \tag{42}$$

and simplifying the resulting expression, we obtain

$$\frac{kT\gamma}{m}=(1/2)(I_o-I+w_o^{\,2}-w^2)-GM_\odot(1/r_o-1/r)+\frac{kT_o\gamma_o}{m}-\frac{1}{4\pi mC}\int_{r_o}^{r}\epsilon(r)4\pi r^2dr \tag{43}$$

where we have written

$$\frac{\mathcal{F}_{rad,\,r_or}}{mNw}=\frac{4\pi\int_{r_o}^{r}\epsilon(r)r^2dr}{4\pi r^2Nmw}=\frac{1}{4\pi mC}\int_{r_o}^{r}\epsilon(r)\cdot 4\pi r^2dr \tag{44}$$

In an entirely analogous manner we may include a term which accounts for the energy deposition, $\Delta(r)$, between r_o and r, giving as a final expression

$$\frac{kT\gamma}{m} = (1/2)(I_o - I + w_o^{\,2} - w^2) - GM_{\odot}(1/r_o - 1/r) + \frac{kT_o\gamma_o}{m} + \frac{1}{4\pi mC}\int_{r_o}^{r}\left[\Delta(r) - \epsilon(r)\right] 4\pi r^2 dr \tag{45}$$

Evaluation of the quantities in equation (45), taking $T = 10^5$ ^oK and $\gamma = 2.5$, shows that $\frac{kT\gamma}{m} \approx 5 \times 10^{13}$ while the radiative loss term has a value of $\approx 2 \times 10^{12}$ if the flux near the earth is taken as 3×10^7 electrons/cm^2-sec. The quantity γ, which indicates the velocity space anisotropy, must satisfy the relationship (Brandt and Michie 1962 eq. (21)) $3/2 \leq \gamma \leq 9/2$. Therefore for all temperatures greater than 10^5 ^oK we can say that the radiative loss term is negligibly small in comparison with the quantity $\frac{kT\gamma}{m}$. Even if the radiative loss must be corrected by a factor $U \approx 3$ this statement still holds. Only when the temperature drops to 10^4 ^oK or less does the radiative loss amount to an appreciable fraction of $\frac{kT\gamma}{m}$; however temperatures as low as this do not occur in the models of the interplanetary gas which we use, and, in fact, a temperature of about 2×10^5 ^oK was found from the Mariner-II results by Neugebauer and Snyder (1962). Thus we may safely assume that $\mathcal{F}_e/\mathcal{F}_m$ is a constant beyond $r = 4R_{\odot}$; this conclusion may be regarded as the most important consequence of the radiative loss calculation. It is still subject to the condition, however, that the energy deposition also be small beyond $4R_{\odot}$. Further calculation is necessary in order to bear out this hypothesis, but if Δ (r) is small it serves to cancel the effects of ϵ (r) and therefore to reinforce the conclusion.

The calculation also has implications regarding the quantity I_o. The expression mw_oNI_o represents the energy flux transported by conduction, evaluated at the point $r = 4R_{\odot}$. This was calculated by Brandt and Michie; they obtained a value of 4×10^2 ergs/cm^2 which they took to be a maximum value, since it included a number of quantities which had not been specifically taken into account, notably, the radiative losses. To obtain consistency of the theoretical model with observation, they found that I_o had to be reduced. This reduction can be accounted for by one or more of the

following effects: (1) coronal magnetic fields which would tend to inhibit conduction; (2) radiative losses beyond $4R_{\odot}$; and (3) an actual particle flux in the solar wind greater than that adopted on the empirical model. As a result of the present calculation, the hypothesis that radiative losses between $4R_{\odot}$ and the orbit of the earth form a significant part of I_o must be discarded. This result is in agreement with the recent rocket results (Neugebauer and Snyder 1962) which give a flux $\approx 1 \times 10^8$ electrons/cm^2-sec. Thus, all or part of the discrepancy is resolved with the new value for the flux[point (3) above]. We still have the possible problem of the coronal magnetic field.

The existence of irregular or non-radial magnetic fields could easily be quite important in determining the thermal conductivity in the corona near $4R_{\odot}$. The thermal conductivity appears to be reduced by a transverse magnetic field by a factor of $(1 + \omega_{ce}{}^2 t_{ce}{}^2)^{-1}$ (Spitzer 1956, Cowling 1954), where ω_{ce} is the cyclotron frequency for electrons and t_{ce} is the collision time for electrons. Thus, for the magnetic field <u>not</u> to be important in inhibiting heat conduction, we must have

$$\frac{eB}{cm_e} \cdot \frac{T^{3/2}}{15N_e} < 1 \quad . \tag{46}$$

When values of the various parameters appropriate to the corona at $4R_{\odot}$ are inserted into equation (46), we find that a transverse magnetic field must be less than 10^{-10} gauss not to inhibit conduction. Since the magnetic field expected in the neighborhood of $4R_{\odot}$ is many orders of magnitude greater than this limit, we conclude that any reasonable <u>transverse</u> magnetic field at $4R_{\odot}$ will seriously inhibit conduction. Thus, a knowledge of the structure of the solar magnetic field at some distance from the sun would seem to be most desirable. We note that the contribution to the radiative loss term from magnetic bremmstrahlung is probably quite small.

It should be mentioned that the low densities found on the semi-empirical model (Brandt and Michie 1962) and the Mariner-II results (Neugebauer and Snyder 1962) are supported by additional evidence. The

low densities are favored on the basis of Wurm's (1962) study of comet comas and on the basis of Öpik's (1956) estimate of the upper limit ($\approx$ 5 electrons/cm^3) of the density of the solar wind consistent with the existence of meteoric particles in the interplanetary medium. The plasma probe measures of Bonetti, Bridge, Lazarus, Lyon, Rossi and Scherb (1962) are also consistent with a low mean density (see Piddington 1962).

Finally, we remark that equation (30) is a general expression which can be used to evaluate the radiative losses in any rarefied gas with a temperature on the order of 10^6 °K in which statistical equilibrium and the Maxwellian velocity distribution hold. In the paper the expression was evaluated over the ranges of temperature and pressure in the outer solar corona; however it could also be applied in other situations. In particular, the spectra of the recurrent nova RS Ophiuchi showed coronal forbidden emission lines in the outbursts of 1933 and 1958, indicating a temperature $\sim 10^6$ °K. Wallerstein (1961) has estimated the electron density after the outburst in the circumstellar region to be on the same order as that of the inner corona of the sun, indicating that the work presented here could also be of value in the explanation of the coronal lines of RS Ophiuchi.

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TABLE 1

Abundances of Various Elements

Element	$\log N_E/N_H + 12$	Element	$\log N_E/N_H + 12$
H	12.0	Si	7.5
He	11.0	S	7.3
C	8.7	Ca	6.2
N	8.0	Fe	6.6
O	9.0	Ni	5.9
Na	6.3	Ne	8.1
Mg	7.4	A	6.3
Al	6.2		

TABLE 2

Ion Densities and Bound-bound Radiative Losses for Heavy Elements

Ion	$p_i = N_i/N_H$	$\lambda(\mathring{A})$	Radiative Loss x 10^{-21} erg/sec.
Mg IX	1.5×10^{-6}	368.1	.25
Mg X	7.5×10^{-6}	625.3, 609.8	.55
Mg XI	1.6×10^{-5}	---+	---
O VI	1.4×10^{-6}	1037.6, 1031.9	.24
O VII	9.7×10^{-4}	21.60	---
Ni XII	1.1×10^{-7}	299, 322	.06
Ni XIII	2.86×10^{-7}	304-332	.10
Ni XIV	2.7×10^{-7}	311-323	.05
Ni XV	1.0×10^{-7}	316, 333, 346	.02
A IX	8.0×10^{-7}	--- --- +	---
A X	9.0×10^{-7}	166, 171	.27
A XI	3.2×10^{-7}	190	.18
C V	3.0×10^{-5}	40.27	.21
C VI	2.5×10^{-4}	33.2	.26
Na IX	2×10^{-7}	681.7, 694.3	.02
Na X	1.8×10^{-6}	--- +	---
Ne VIII	2.5×10^{-6}	792.5, 782,4	.25
Ne IX	1.2×10^{-4}	--- +	---
Al X	4.8×10^{-7}	332.9	.08
Al XI	7.0×10^{-7}	550.0, 568.5	.04
Si IX	4.7×10^{-6}	342-350	.42
Si VIII	4.7×10^{-7}	314, 316, 320	.04
Si X	1.38×10^{-5}	356.1, 347.4	1.05
Si XI	1.38×10^{-5}	303.6	1.76
Si XII	4.5×10^{-6}	521.1, 499.3	.27
S VIII	6.3×10^{-7}	198.6, 202.6	.14
S IX	4.0×10^{-6}	222-228	.56
S X	9.8×10^{-6}	258, 260, 264	.64
S XI	6.2×10^{-6}	290	.36
S XII	6.0×10^{-7}	302, 290	.03
Fe XII	3.5×10^{-7}	356, 360, 369	.07
Fe XIII	9.6×10^{-7}	360, 373, 386	.20
Fe XIV	1.8×10^{-6}	346, 370	.37
Fe XV	1.2×10^{-6}	288	.44
Fe XVI	2.6×10^{-7}	361.7, 336.2	.04
N VI	7.5×10^{-5}	--- +	---
Ca XI	1.5×10^{-6}	--- +	---
TOTAL			9.0×10^{21} erg/sec

X-Ray Lines Included in Calculations

Element	$\lambda(\mathring{A})$
C IV	312
N V	210
N VI	29
N VII	19
O VI	150
O VII	22
Ne VII	95
Ne VIII	85
Mg VIII	75
Mg IX	63
Mg X	58
Si VIII	67
Si IX	55
Si X	51
Si XI	42
S VII	70
S VIII	60
S IX	52
S X	43
Fe IX	150
Fe X	97
Fe XI	88
Fe XII	73
Fe XIII	65
Fe XIV	62
Fe XV	55
TOTAL	26.2×10^{21} erg/sec

no lines calculated for this element

TABLE 3

Contributions of the Various Processes to the Radiative Loss (equation 1)

(Unit: 10^{21} erg/sec)

Hydrogen, total	3.5
Free-free	1.8
Free-bound	1.1
Bound-bound (upper limit)	0.6
Helium, total	4.3
Free-free	0.7
Free-bound	1.8
Bound-bound (upper limit)	1.8
Heavy elements, total	41.8
Free-free	0.1
Free-bound	1.9
Forbidden lines	0.1
Permitted lines+	9.0
(upper levels)*	4.5
X-ray lines	26.2
Total	49.6

+For details see Table 2

*See (4) under results.

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